EE 2030 Linear Algebra Chi-chao Chao
Spring 2010

Homework Assignment No. 1
Due 10:10am, March 17, 2010

`Reading:` Strang, Chapters 1 and 2.

`Problems for Solution:`

1. (a) Find the pivots and the solutions for both systems:

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}.$$

   (b) If you extend (a) following the 1, 2, 1 pattern or the −1, 2, −1 pattern, what is the fifth pivot? What is the nth pivot?

2. Find the triangular matrix $\boldsymbol{E}$ that reduces "*Pascal matrix*" to a smaller Pascal:

$$\boldsymbol{E} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}.$$

   Which matrix $\boldsymbol{M}$ reduces Pascal all the way to $\boldsymbol{I}$?

3. Find $\boldsymbol{A}^{-1}$ and $\boldsymbol{B}^{-1}$ (if they exist) by the Gauss-Jordan method:

$$\boldsymbol{A} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$

4. (a) Find $\boldsymbol{A}^{-1}$:

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

   (b) Extend $\boldsymbol{A}$ in (a) to a 5×5 "alternating matrix" and guess its inverse; then multiply to confirm.

5. *Tridiagonal matrices* have zero entries except on the main diagonal and the two adjacent diagonals. Factor these into $\boldsymbol{A} = \boldsymbol{LU}$ and $\boldsymbol{A} = \boldsymbol{LDL}^T$:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{A} = \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix}.$$

6. If $\boldsymbol{A} = \boldsymbol{L}_1\boldsymbol{D}_1\boldsymbol{U}_1$ and $\boldsymbol{A} = \boldsymbol{L}_2\boldsymbol{D}_2\boldsymbol{U}_2$, where the $\boldsymbol{L}$'s are lower triangular with unit diagonal, the $\boldsymbol{U}$'s are upper triangular with unit diagonal, and $\boldsymbol{D}$'s are diagonal matrices with no zeros on the diagonal, prove that $\boldsymbol{L}_1 = \boldsymbol{L}_2$, $\boldsymbol{D}_1 = \boldsymbol{D}_2$, and $\boldsymbol{U}_1 = \boldsymbol{U}_2$.

   (a) Derive the equation $\boldsymbol{L}_1^{-1}\boldsymbol{L}_2\boldsymbol{D}_2 = \boldsymbol{D}_1\boldsymbol{U}_1\boldsymbol{U}_2^{-1}$ and explain why one side is lower triangular and the other side is upper triangular. (You may use the fact that the $\boldsymbol{L}$'s and $\boldsymbol{U}$'s are invertible.)

   (b) Compare the main diagonals in that equation, and then compare the off-diagonals.

7. If $\boldsymbol{A}$ and $\boldsymbol{B}$ are symmetric matrices, which of these matrices are certainly symmetric? (You need to justify your answer.)

   (a) $\boldsymbol{A}^2 - \boldsymbol{B}^2$.

   (b) $(\boldsymbol{A} + \boldsymbol{B})(\boldsymbol{A} - \boldsymbol{B})$.

   (c) $\boldsymbol{ABA}$.

   (d) $\boldsymbol{ABAB}$.

8. Factor the following matrices into $\boldsymbol{PA} = \boldsymbol{LU}$. Also factor them into $\boldsymbol{A} = \boldsymbol{L}_1\boldsymbol{P}_1\boldsymbol{U}_1$.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 1 & 1 & 3 \end{bmatrix} \quad \text{and} \quad \boldsymbol{A} = \begin{bmatrix} 0 & 4 & 5 \\ 0 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}.$$